The Incomplete Story of Łukasiewicz and Bivalence*

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Jan Łukasiewicz is first and foremost associated with the rejection of the principle of bivalence and the discovery of many-valued logics. The story of both the rejection and the discovery is, however, incomplete in many senses, the most obvious being that it has not been told in its entirety. There are specific questions, especially regarding the first period of Łukasiewicz's activity, which have not yet found a clear and convincing answer in the critical literature. Łukasiewicz's rejection of the principle of the excluded middle in 1910, according to which the principle fails in application to general objects, as well as (probably) to future ones, has more than once been confused or deliberately assimilated with his later rejection of bivalence.[1] Consider the following questions: (i) did Łukasiewicz detect in 1910 the link between determinism and the principle of bivalence? (ii) did he have the means, at the time, to distinguish between the latter and the principle of the excluded middle, and could he thus reject excluded middle but not bivalence? If in 1910 he just rejected the former, (iii) from when did he reject the latter? Failure to properly address such questions compromises a proper reconstruction of Łukasiewicz's thought, to which this paper aims to contribute. By answering (i), (ii), (iii), on the basis of the few sources on the excluded middle at our disposal, backed up by an interpretation by analogy of *On the Principle of Contradiction in Aristotle*, I aim to offer a fairly satisfactory account of Łukasiewicz's early ideas in this respect. In the following I argue specifically (i´) that Łukasiewicz did not detect the link

[1] My conclusions in this paper are at odds with parts of Simons (1989), which gives an account of Meinong's influence on Łukasiewicz, and of the latter's relevant views before 1917 (cf. in particular p. 197) and Woleński & Simons (1987). I agree with Woleński (1987), p. XXXVI fn22 that "there is [...] no basis to think that already at that time Łukasiewicz questioned the principle of bivalence", and that Greniewski (1957), p. 94, is wrong in writing that Łukasiewicz at the time "felt somewhat ill at ease, maybe even foreign in the bivalentists' camp". English translations from Polish are mine unless otherwise indicated.

between determinism and bivalence in 1910; (ii´) that he had the tools to distinguish between bivalence and excluded middle; and (iii´) that he rejected bivalence only in 1917. In the literature the first two questions are not dealt with in a clear way.[2] As regards the third, strictly connected with the other two, 1917 is apparently agreed upon as the date of the construction of the first system of three-valued logic, and therefore as the date of the introduction of the third value. Yet it is suggested here and there that we should look for the rejection of bivalence in Łukasiewicz's earlier writings. I think this is not right, and that, instead, a correct chronology of Łukasiewicz's writings should rely on his rejection of bivalence in 1917.

Apart from specific ones, however, there are also more general reasons for taking an interest in Łukasiewicz's early writings (i. e., before 1917). Those works are enjoying considerable attention from scholars interested in the origins of paraconsistent logics,[3] as Meinong-inspired contradictory objects feature in Łukasiewicz's extensive attack upon the principle of contradiction, which he granted only limited applicability. It is then most appropriate to get an adequate analysis of Łukasiewicz's early ideas and to see whether they are still of some use. I think it is rather doubtful that his views can be used to argue against either bivalence or requirements of non-contradiction for objects, but this must be first shown on a textual basis. The early Łukasiewicz is, unfortunately, a hard nut to crack. If Meinong's jungle is one to defoliate, then so is Łukasiewicz's. And here the language barrier plays, as usual, a disgraceful role. Making the Polish analytic heritage an accessible source of philosophical reflection still seems to be a quixotic idea.[4] Another reason for caring about the early Łukasiewicz is that the issues mentioned above involve topics still being discussed. One is term negation;[5] another is the application of the latter to sentences whose subjects (are taken to) denote a particular kind of non-existing objects, general and future objects, or *incomplete objects*, as Łukasiewicz termed them following Meinong – a second sense in which the story of Łukasiewicz and bivalence is 'incomplete'.[6] And, finally, Łukasiewicz's early activity, characterized by object-theoretical reflections in the fashion of Twardowski and Meinong, and imbibed also with a significant Bolzanian influence, is of

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[2] Unclarities and misunderstandings similar to the ones discussed in this paper regarding Łukasiewicz's stand have arisen also with respect to Peirce, cf. Lane (2001), (1997).

[3] See Priest & Routley (1989), pp. 25-29; Arruda (1989), p. 101; Da Costa, Béziau & Bueno (1995), p. 598; Poczobut (2000), p. 12.

[4] Something is slowly beginning to happen, though: a collection of Łukasiewicz's writings in Polish, including very small (and very interesting) pieces appeared in 1998, thanks to the editorial effort of Jacek J. Jadacki, cf. Łukasiewicz (1998). This edition has allowed some parts of the present paper to be more accurate or, in some cases, possible in the first place.

[5] Cf. for instance LaPalme Reyes, Macnamara, Reyes & Zolfaghari (1994).

[6] Modern cognates of general objects are Kit Fine's arbitrary objects (cf. Fine (1985) pp. 5 and *ff.* pp. 44-5) and Terence Parsons' incomplete objects (cf. Parsons (1980), p. 19).

importance to historians, not least because it is not yet marked by Łukasiewicz's later metalogical interests, which played a substantial role in the metalogical turn in Poland as a whole.

I. Bivalence, ontological *Tertium non datur* and logical *Tertium non datur*

In a talk on the principle of the excluded middle from 1910, Łukasiewicz claimed that the principle of the excluded middle fails to obtain in application to general objects and (probably) to future objects.[7] In his paper on Łukasiewicz and Meinong Peter Simons observes that this means that (a) Łukasiewicz had detected as early as 1910 the link between determinism and bivalence – which is notoriously Łukasiewicz's reason for rejecting classical logic[8] – because (b) he and others did not distinguish at that time bivalence from the excluded middle. These claims seem to suppose that we should take Łukasiewicz's 1910 excluded middle to be

(1) $\alpha \vee \neg \alpha$

where α is a schematic letter standing for sentences. If this were Łukasiewicz's excluded middle, it would be true that neither he nor his contemporaries could actually distinguish (1) from the metalogical principle of bivalence (metalogic in the usual sense was still to come).[9] However, what Łukasiewicz had in mind when speaking of the excluded middle was not (1).

Here three things are at issue: the principle of bivalence, the *ontological* principle of the excluded middle and the *logical* principle of the excluded middle. In his monograph *On the Principle of Contradiction in Aristotle* (1910) Łukasiewicz was the first in the Polish tradition to clearly distinguish between ontological and logical principles.[10] By analogy with Łukasiewicz's distinction between the different versions of the principle of contradiction, which he elaborated with reference to Aristotle, the two versions of the principle of excluded middle, ontological and logical, can be reformulated as follows:

OTND   Every object *a* has either the property *b* or else non-*b*;

LTND   Two sentences of the form, respectively, 'the object *a* has the property *b*' and 'the object *a* has the property non-*b*' are not both false at the same time.

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[7] Cf. Łukasiewicz (1910b).
[8] Cf. Łukasiewicz (1918) and (1922/3).
[9] Cf. DeVidi & Solomon (1999) on the confusions about bivalence and the excluded middle.
[10] Cf. Łukasiewicz (1910a), pp. 9 and *ff*; Germ. transl. p. 9 and *ff*.

Łukasiewicz considered OTND and LTND to be *equivalent*.[11] These formulations, in a somewhat more ambiguous and general form, became also standard in the Polish milieu for some time, being employed by Leśniewski in particular.[12] Another encoding of the principle, proposed by Łukasiewicz in the Appendix to the *Principle of Contradiction in Aristotle*, is

(2) $a+a' =1.$

Principle (2), written in the style of Louis Couturat's *L'Algèbre de la logique*, reads: "either the object *a* has the property *b* or the object *a* does not have the property *b* iff *a* is an object".[13] The fact that (2) has this particular biconditional formulation is irrelevant for our purposes here.[14] The crucial point instead is that $a+a'$ in (2) is just another way of writing OTND: the negation a' in (2) is not the *sentential* negation most philosophers are familiar with, that of (1), but *term* negation, also called *nominal* or *predicate term* negation.[15] If we disregard that Łukasiewicz adopts the Bolzanian form of truth-bearers, i. e. '*a* has *b*-ity' (e. g., 'Tommy has happiness'), and not the traditional '*a* is *b*' (e. g., 'Tommy is happy'), an uninteresting difference here, we can observe the following. In Leśniewski's theory of semantic categories, where with *s* and *n* the two basic categories of *s*entences and *n*ames are indicated, '¬' in (1) is a sentence-forming functor of one sentential argument (category: *s/s*), while the negation in (2), which could be described rather as a negative copula 'is-not', is a sentence-forming functor which takes two names as arguments (category: *s/nn*). In the context of the present discussion '*a* is-not *b*' is equivalent to '*a* is non-*b*', where the nominal negation 'non-' is a name-forming function taking a name as an argument (category: *n/n*). It is *not*, however, equivalent to 'not: *A* is *b*' (¬). As is usually the case in Łukasiewicz's early writings (and in Leśniewski's), the

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[11] Cf. Łukasiewicz (1910a), p. 149: "for true judgments, positive and negative, correspond to objective facts, that is to the relations of possessing and not possessing a property by an object"; Germ. transl. p. 183. In this work judgments are linguistic items.

[12] Cf. for instance Leśniewski (1912), §1, p. 203 (Eng. transl. p. 21) and Leśniewski (1913), §1, p. 316 (Eng. transl. p. 48). Note however that for Leśniewski OTND and LTND are *not* equivalent: the first is true, the second false.

[13] Łukasiewicz (1910a), pp. 155 and 170-1; Germ. transl. pp. 189, 212 and *ff.*

[14] This is linked to a difference Łukasiewicz detects between a *formal* and a *concrete* proof of fundamental logical principles like the principle of contradiction, cf. Łukasiewicz (1910), p. 152; Germ. transl. p. 186. Cf. also Betti (200X) and Raspa (1999).

[15] The distinction "comes in a discouraging array of guises", cf. the impressive chart in Horn (1989), p. 140. Horn's book is an indispensibile source of infomation on the topics discussed in this section. 'Predicate negation' and 'predicate term negation' do not always stand for the same thing. For 'nominal negation', cf. Henry (1972), p. 38; Wessel (1982), p. 1363, speaks of an *Absprechenoperator*. A parallel with von Wright's strong and weak negation (1959) is probably appropriate, cf. Horn (1989), p. 132 and *ff.*

negation in sentences of the form '*a* does not have *b*' is term negation, so that such sentences are in fact to be understood as having the form '*a* has non-*b*'. Sentential negation is instead normally expressed by the clause 'is false'.[16] Incidentally, notice that LTND concerns only *singular* sentences. For when Łukasiewicz originally formulates the logical principle of contradiction he writes:

> two judgments of which one ascribes to *an object* [*przedmiotowi*] just the property which the other denies *to it* [*mu*] cannot be true at the same time.[17]

We obtain the logical principle of the excluded middle just by substituting 'false' for 'true' in the passage above. Although it is enough for the present purposes to take into account only pairs of singular sentences, LTND could easily be reformulated in order to hold also for quantified sentences. What is important is that the negation is understood as term negation and not according to the standard matrix of sentential calculus. Such reformulation of LTND would apply, for instance, to the following pair of sentences "All triangles are equilateral" and "Some triangles are non-equilateral".

In sum, neither (2), OTND nor LTND are equivalent to (1). As will be clearer in the following, it is precisely because of the way Łukasiewicz understands negation in OTND/LTND that bivalence is not at all what is involved in his rejection of the principle of the excluded middle. Consider

BIV   There are exactly two truth-values: true and false.

Is the one who rejects OTND/LTND compelled to reject BIV? Definitely not. Moreover, things do not change if one strengthens BIV by including also the determinacy claim

DET   Every sentence has exactly one truth-value

in order to obtain

BIV*   Every sentence is either true or else false.

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[16] In the present context term negation is also not distinct from *privation*. Cf. also Bolzano (1837) §127, II 15: the negation of a property *b* is formulated as 'lack of *b*(-ity)', as in '*a* has lack-of-*b*(-ity)', while the negation of a proposition '*a* has *b*-ity' corresponds to '*a* has lack-of-truth' – where lack-of-truth is taken to be falsity.

[17] Cf. Łukasiewicz (1910a), p. 11, my emphasis; Germ. trans. p. 11.

Even if Łukasiewicz's final 1917 position was such as to reject BIV* (or rather keeping DET and rejecting BIV), we cannot say that he did this already in his 1910 talk on OTND/LTND. There Łukasiewicz claimed that the excluded middle (more precisely OTND) does not apply to *general objects* like, for instance, the triangle in general. Further, he stated that the principle of the excluded middle (more precisely LTND) does not apply to sentences whose subjects denote such objects. The essential point here is that he also maintained that "if one denied that future phenomena are already now in every respect determined, probably one could not accept the principle of the excluded middle".[18] This, however, has nothing to do with failing to distinguish between BIV* and either OTND or LTND, or both, and *therefore* rejecting BIV*. Consider the following sentences:

(1)   The triangle (in general) is equilateral
(2)   The triangle (in general) is non-equilateral
(3)   The next Prime Minister of Italy will be a Milanese
(4)   The next Prime Minister of Italy will be a non-Milanese.

The objects denoted by the subjects of (1)-(2) are considered by Łukasiewicz to violate OTND. A triangle in general is neither equilateral nor not-equilateral, simply because it is not defined with respect to the property of equilaterality:

> the triangle in general is indeed defined with respect to the quantity of its sides, because this property is essential to all triangles; it is not, however, defined with respect to the accidental properties of equilaterality and non-equilaterality;[19]

and therefore

> both the judgments 'the triangle is equilateral' and 'the triangle is not equilateral' appear to be false.[20]

This is why LTND does not apply to them. Analogously, if future objects are (*now*, incidentally) not fully determined, the objects denoted by the subjects of (3) and (4) also violate OTND, and therefore LTND fails to apply to the pair (3) and (4) as well. But clearly, infringing LTND is not tantamount to infringing BIV*; in particular you would reject DET if you stated that the pairs

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[18] Łukasiewicz (1910b).
[19] *Ibid.*; Eng. transl. here quoted with changes.
[20] *Ibid.*; Eng. transl. here quoted with changes.

(1)-(2) and (3)-(4) are neither true nor false (in a semantics with truth-value gaps, for instance), or reject BIV if you stated that they possess a third value.[21] Also, to properly state BIV* one does not need anything more than taking truth and falsity as properties of sentences, which are particular objects, and BIV* turns out to be the following *special case* of OTND:

OTND*　　Every sentence has either the property *truth* or else *non-truth,*

where *non-truth* equals *falsity.*

II. Yes, but what about probability?

In his 1913 paper on probability, however, Łukasiewicz claimed quite explicitly that there are some sentences which invalidate OTND* (BIV*). But Łukasiewicz is likewise explicit on this: the sentences infringing BIV* are *not* sentences like (1)-(4). Łukasiewicz considers two kinds of sentences: *determined sentences* (also called *judgments*), and *undetermined sentences*. Undetermined sentences are a quite peculiar kind of sentences containing a free variable, like

(5)　　*x* is an Englishman.

　　Sentences like (1)-(4) are instead determined sentences, as they do not contain any variable, like 'W. H. Auden is an Englishman'. Sentences like (5) are only apparently what we now call 'open sentences'.[22] They are rather thought to function like Bolzanian propositions-in-themselves in which some ideas-parts *vary* according to a certain rule (read: have a domain of variation), and so generating *many* other propositions of the same kind. For Bolzano the *degree of validity* (*Gültigkeit*) of (5) is 1 if and only if all the propositions generated from (5) by varying the subject-idea (in this case: the place-holder *x*) are all *true*; it is 0 if and only if all the propositions generated from (5) by varying the subject-idea are all *false*; and it is an intermediate value between 0 and 1 if from (5) true as well as false propositions are generated. The validity degree (for Łukasiewicz: *logical value*) is thus given by the *ratio* of all true propositions which are generated from a given proposition to all propositions that can possibly be generated from it.[23] The reason why determined sentences in Łukasie-

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[21] I am presupposing here a clear difference between a sentence getting a truth-value other than true or false and a sentence not getting any ('truth-valueless'), but the issue of what a truth-value gap would exactly be does not seem uncontroversial. Cf. Glanzberg (200X), p. 3.

[22] Cf. Childers & Majer (1998), p. 303.

[23] Cf. Bolzano (1837), §147. Cf. also Jan Berg's introduction to *Bernard Bolzano-Gesamtausgabe*, I/12, 1, p. 55.

wicz's view are just true or false and do not possess an intermediate value seems to be that for them there is no *ratio* that can be established in this way. It is therefore particularly important to stress that for Łukasiewicz *determined* statements about future events *are* (now) either true or false, even if *before the event* we cannot know which ones are true and which ones are false:

> It can no longer be asserted that such events or sentences as 'This die will now turn up 6' or 'the next drawing from this urn will yield a black ball' are probable. Such sentences, being *determined* judgements, are either true or false, even if *before* the event we can never know *which* of them are true and which are false.[24]

This tells us how far Łukasiewicz was at this point from his later approach linking future contingents and a third kind of (determined!) sentences. First, the difference between determined and undetermined sentences in the sense explained above ceased to play any role in his post-1917 works. Secondly, we can easily modify OTND* substituting 'sentence' by 'determined sentence' in order we picture better Łukasiewicz's views in 1913:

OTND** Every determined sentence is either true or false.

Now it is easy to see that Łukasiewicz accepted OTND** up to 1917, but it is exactly OTND** which he later rejected. The picture at this point can be summed up as follows. If future objects enjoy the same status of general objects, i. e., are as undetermined as to some properties as general objects are, then (as stated in 1910) both

(6)   'the next ball to be extracted from this box will be black'

and

(7)   'the next ball to be extracted from this box will not be black'

are false. This is a counterexample to LTND, as negation in (7) is understood like in (4) as predicate negation. Still, this would have nothing to do with reject-ing BIV*, because (6) and (7) would not have a third value. It is clear that at this particular time of Łukasiewicz's development no determined sentence can be neither true nor false. As this is stated also in his 1916 paper on the concept of magnitude, which fully squares with Łukasiewicz's ideas on probability,[25] it

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[24] Łukasiewicz (1913), p. 34; Eng. trans. p. 38, here quoted with slight changes.
[25] Cf. Łukasiewicz (1916), p. 311; Eng. trans. p. 71.

turns out that 1917 can indeed be used to establish a chronology.[26] We have no reasons to be puzzled by the fact that later, in 1930, Łukasiewicz played down any connection between the rejection of bivalence and his 1913 theory of probability.[27] Further, if it were *per absurdum* true that Łukasiewicz introduced the third value in connection with his 1913 paper on probability, then we would be fully justified in claiming that the third value was present in 1909 already, since a short piece from 1909 shows that Łukasiewicz's 1913 ideas on probability were already ripe in that year.[28] This latter circumstance, incidentally, also weakens the hypothesis of an influence on Łukasiewicz of Meinong's theory of probability.[29] And it can be shown on the basis of a textual analyses that the strongest influence on Łukasiewicz's theory of probability was rather exerted by Bernard Bolzano.

III. Objection!

Some may still not feel convinced by the story as I tell it. There is indeed at least one passage from 1910 in which Łukasiewicz says that statements about general objects are neither true nor false:

> Let us consider the object "column in general" without any further definition. [...] Should we consider a judgment such as 'the column is sharp' true or false? [...] the judgment 'the column is sharp' is neither true nor false.[30]

This passage comes from the book on contradiction and was written *before* the 1910 talk on OTND/LTND. Should we then think that roughly in the same period Łukasiewicz thought that statements about general objects are false (talk) and that they are neither false nor true (book), and therefore that after all he did *not* distinguish OTND/LTND from BIV*? No, we should not. Compare the footnote attached to the passage quoted above:

> I accept here for the moment Meinong's view. However, a question arises: should we not consider judgments of this kind, like "the column is sharp", "the column is not sharp", "the triangle is equilateral", "the triangle is not

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[26] *Contra* Sobociński (1956/7), p. 29: "in the scientific activity of Prof. Łukasiewicz one can distinguish two periods: the Lvov period, lasting until 1915, and the Warsaw period, starting from that year".

[27] Cf. Łukasiewicz (1930), p. 113; Eng. trans. p. 173.

[28] Cf. Łukasiewicz (1909), p. 231.

[29] Simons (1989), p. 204, offers reasons *pro* and *contra*, but in his diary Łukasiewicz denied Meinong's influence, cf. Łukasiewicz (1949/54), p. 61-61v.

[30] Łukasiewicz (1910a), p. 113; Germ. transl. p. 138.

equilateral" and so on, false anyway? This question is linked with the prin-
ciple of the excluded middle, which constitutes, as is known, a counterpart
to the principle of contradiction. Should the judgments mentioned above
be considered false, it would be a characteristic of incomplete objects that
they do not fall under the principle of the excluded middle.[31]

Łukasiewicz, therefore, clearly detects the position in the passage, the one
according to which sentences like (1) and (2) are neither-true-nor-false, as
Meinong's, not as his own. Łukasiewicz instead states his position in the foot-
note. He simply does not specify his views in detail, but what he already men-
tions in the footnote he was to state more firmly in the talk on the excluded
middle. It is not difficult to tell why Łukasiewicz found it convenient to take
sides with Meinong, as the mention of incomplete objects is fully Meinongian.
Łukasiewicz, however, adds some remarks of his own as well: the triangle in
general is called, both in 1906 and in the book on contradiction, an *abstract
ideal object*, a *construction* or an *object of construction*, while the column in gen-
eral is called an *abstract real object*, a *reconstruction* or an *object of reconstruc-
tion*.[32] Next to general objects like the column in general there are also concrete
objects, like the column Mickiewicz in Lvov. The salient difference between
concrete and abstract objects is what Łukasiewicz and Meinong would term a
difference in *completeness*. A concrete object is a complete object, that is to say,
OTND holds. Complete objects are *things*, *properties*, *phenomena*, *events*: the
Eiffel Tower is a complete object as it is determined in its most minute details.
This is not the case with abstract objects (both real and ideal), which Łukasie-
wicz temerariously enough also calls *concepts*, which are, therefore, incomplete
objects. To *constructions* like the triangle in general nothing *corresponds* in the
external world, while to *reconstructions* like the tower in general corresponds
something in the external world (namely particular colums). According to
Łukasiewicz, reconstructions are of use in science exactly like concrete objects.
Biochemistry and sociology do not just investigate particular nerve cells and
revolutions, but also nerve cells and revolutions *in general* to which particular
cells and revolutions correspond. These general objects are complete with
respect to some properties, but incomplete with respect to other ones, which
could namely be termed *essentially complete* and *accidentally incomplete*. A recon-
struction like the king in general has essentially a kingdom, but need not be a
particular one; on the contrary, if you ascribe to the king in general a particu-
lar kingdom, say, Spain, then you get a king *of Spain* in general, but not a king
in general anymore. So it is false that the king in general has the property of
being a king of Spain. But given this, you are not entitled to ascribe to the king

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[31] *Ibidem*, footnote.
[32] Cf. Łukasiewicz (1906), p. 15; Łukasiewicz (1910a), p. 114; Germ. transl. p. 140.

in general the property of *not being a/the king of Spain* in the sense of OTND, because this would mean ascribing him a particular kingdom, say Sweden or Morocco, and the king in general is a general one exactly because it is not determined as to which kingdom he has, the number of hair he has on his head and so on. OTND is to be understood as saying this: for every property of a certain species, every object *a* has one of them (I here leave out what exactly a species would be).

IV. Why did Łukasiewicz change his views?

The introduction of the third value as in the first system of 1917, not to mention the metalogical way in which Łukasiewicz deals with it in the later three-valued system of 1920 (possibly identical to the former),[33] had not yet been made in 1913. Before 1917 Łukasiewicz was still, though questioning OTND and LTND, inside bivalence. In 1913 he was even convinced that if a sentence is undetermined and probable at any certain given time-point *t*, it is *eternally* probable, i.e., probable at any future time-point t_1 following *t*. But at a certain point Łukasiewicz abandoned his previous ideas, starting to question bivalence for determined sentences, and firmly rejecting the eternity conception for *possible* ones (as he sometimes liked to call the third kind of sentences). What made him change his views? Since he already knew Meinong's positions, we might assume he could have realized only later the rich implications of Meinong's views. Other hypotheses are possible, though. The most probable is that Łukasiewicz's turn was a result of the 1913 controversy about eternity and sempiternity of truth between Leśniewski and Kotarbiński. That Kotarbiński's perspective had an influence on Łukasiewicz's rethinking of the semantics of future contingents is something that scholars agree upon.[34] Kotarbiński made the necessary step towards having another kind of sentences next to true and false, the ones he called *indefinite* and whose subjects denote what would be perfectly determined objects if they were present. The sole odd detail is that Łukasiewicz in 1916 is still inside bivalence (but so was Kotarbiński, apparently). Only from 1917 onwards were sentences of whatever kind denoting a *possible* object (something that could be actualised only in the future) associated with a third value. The reason why Łukasiewicz's turn took place only later might depend on the fact that this new standpoint required a change in perspective as to what is involved in a sentence's truth-making. Up to 1917 Łukasiewicz never linked undetermined statements with direct 'undeterminedness-making', so to speak: in his theory of probability some sentences got truth-

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[33] Simons (1989), p. 200, raises the question whether these are two systems or one.
[34] See the discussion in Woleński (1990), p. 195.

values different from true and false as a result of their being related to other sentences, not as a result of an evaluation based on their truth-maker. But in Kotarbinski's paper indefinite sentences are such because they are about unde-termined, i. e., *possible*, objects.[35] Although Łukasiewicz would also later speak about *possibilia*, the turn immediately preceding the construction of the first three-valued system was probably also the last occasion on which Łukasiewicz was specifically concerned with the theory of objects. In particular, although his main work in the early period was concerned with a criticism of the prin-ciple of contradiction on an object-theory basis, starting from 1910 onwards Łukasiewicz became less and less concerned with fighting the principle of which the excluded middle was a counterpart, the principle of contradiction in its ontological and logical versions.

Łukasiewicz's switch from ontology to logic and metalogic is in general very clear to see: it must be seen as associated with the dissemination of the sentential calculus in Poland that took place during the First World War. Woleński is right that Łukasiewicz's courses in logic before 1917 cannot have exceeded the level of Couturat's book on the algebra of logic.[36] There is a quo-tation in print from Łukasiewicz's diary in which he says that although he start-ed lecturing on logic in 1906, he occupied himself with mathematical logic only later and between the wars – and he is obviously speaking of his post-1917 systems.[37] Now, when Łukasiewicz rejected bivalence metalogically, i. e., insert-ing into the logical matrix of the bivalent propositional calculus the value '2', he obtained a trivalent matrix for a trivalent calculus in which both (1) and

(8) $\neg(a \wedge \neg a)$

got the third value for *a* getting the third value. Until the rejection of bivalence Łukasiewicz was only able to challenge the principles of contradiction and excluded middle, instead, on the basis of a Meinong/Twardowski-like theory of objects *and* on the basis of a logic of terms equipped with term negation. Without this latter (with the sentential negation of (1) and (8) in its place, instead) he could not reject OTND/LTND without stepping outside the frame-work of BIV*. It is no accident that Łukasiewicz was the historian of logic to point out that Aristotle has a logic of terms and the Stoics a logic of proposi-tions. He himself lived through this very same change of paradigm. The switch was made possible by Łukasiewicz's acquaintance with Russell and Frege, but it was also immediately connected with an understanding of the 'new logic' as the study of logical systems and their metalogical properties. These circum-

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[35] Cf. Kotarbiński (1913), p. 88.
[36] Cf. Woleński (1995), p. 65.
[37] Cf. Łukasiewicz (1949/54), p. 60.

stances are obviously of interest, as the switch from the Unique Logic in use to the many logical systems as (basically inert) objects of investigation did not merely concern Łukasiewicz, but was a major event in the history of logic.

The reason why Łukasiewicz stopped doing ontology and did not even develop a calculus of names like Leśniewski's is probably linked with the difficulties posed by his particular theory of objects and concepts, and a probable (consequent) lack of interest. It is vital to notice that all what has been said so far about Łukasiewicz's general objects presupposes the hazardous claim that reconstructions do actually possess properties which are of the *same ontological type* as the concrete ones which they are reconstructions of: the tower in general simply lacks many properties which the Eiffel Tower has (being in Paris, being made of iron, etc.), i. e., has all the essential properties for being a tower, but none of the accidental properties. Łukasiewicz seems to think that it is always possible to obtain from any given abstract real object a corresponding concrete one by adding properties. This is just what distinguishes ideal from real abstract objects, and it means: if I take a reconstruction like the tower in general I can complete it by adding to it the accidental properties it lacks. But if I take a construction like the triangle in general I cannot add to it any property to get some concrete object. One may well fear that there must be something terribly wrong with this, but this seems to be just what Łukasiewicz had in mind. No wonder that Leśniewski's 1913 proof against general objects is indeed effective only against theories conflating types of properties, like Łukasiewicz's, despite the fact that Leśniewski assigns a far greater relevance to it.[38]

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[38] Cf. Leśniewski (1913), §1, *Remark* 2, pp. 318 *ff*; Eng. transl. p. 50 and *ff*. Cf. also Betti (200X).

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